Scheduling Smart Home Appliances in the Stockholm Royal Seaport

Degree project in
Automatic Control
Master’s Degree Project

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Abstract

This thesis investigates the optimal scheduling of smart home appliances with respect to economic benefits (electricity bill) and reducing environmental impacts (CO₂ emissions) for the Stockholm Royal Seaport project. The aim of this project is to develop a new urban district developing in the eastern Stockholm which will house 10,000 new apartments and 30,000 new office spaces where modern living is combined with environmental thinking to create sustainable living. In a previous work the scheduling objective was to minimize electricity bill, subject to various constraints such as sequential processing and consumer preferences. In this work the optimization framework will be extended to consider the trade-off between electricity bill and CO₂ emission minimization. This is a main concern in the Royal Seaport project. The study of this thesis shows that a well balanced result between minimizing the electricity cost and reducing the CO₂ emissions for an unusual cold day in Sweden (2010-01-05) with three typical home appliances showed that for formulation suggested in a previous work one could save up to 35.9% of electricity costs as well as reducing the CO₂ emissions with up to 16.5%. This saving is with respect to the worst case scheduling for that specific day. The trade-off analysis is based on multi-objective Pareto frontier exploration, which requires solving multiple schedules instead of one as in the previous single objective case. In addition, for practical implementation the smart home control devices that will be used in the Stockholm Royal Sea project apartments will have a CPU and memory similar to those of a smart phone. Therefore there exists a need for faster implementation. The studies in this thesis indicate that the proposed simplified formulation can lead to a almost sixfold speed up in solve time, while providing schedules similar to those by the previous approach. The solve time of the proposed formulation decreases when the number of breakpoints in the piecewise linear objective decreases. A variant of the Ramer-Douglas-Peucker algorithm is applied to reduce the number of breakpoints while guaranteeing the objective function error is within a pre-specified bound. Finally, extensions of the current framework are discussed.
Acknowledgements

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This thesis presents a Master of Science degree project conducted at the Automatic Control lab, school of Electrical Engineering at the Royal Institute of Technology, Stockholm, Sweden. This thesis investigates the scheduling of smart home appliances to achieve the minimum electricity costs and CO$_2$ emissions for the Stockholm Royal Seaport project. The Stockholm Royal Seaport project aims to develop environmentally friendly urban district in the eastern Stockholm which encloses an area of 236 hectares. At the project completion year 2025 it will house 10,000 new apartments and 30,000 new office spaces [1].
Chapter 2

Introduction

Price and environmental awareness continues to rise. There exists a huge interest to live as economical as possible but at the same time environmentally friendly. The fact that the Stockholm Royal Seaport project [1] is about modern living in combination with environmental thinking is a proof of the increased awareness. All these is possible in a smart home, which is a house that has advanced automatic control systems for different processes such as lighting, temperature and smart home appliances. Smart home appliances utilize computer and communication technology to take advantage of an energy smart grid which is an intelligent electricity network that integrates the actions of all users connected to it.

This thesis introduces and evaluates two formulations to schedule smart home appliances with respect to economic benefits and environmental benefits. In a previous work [2], the results showed that the solve time increases rapidly for the scheduling of the smart home appliances as the number of appliances increased. The smart home control devices that will be used in the Stockholm Royal Sea project apartments will have a CPU and memory similar to those of a smart phone. In addition to that the algorithm should be able to compute a set of different choices such as economic choices, environmental choices and balanced choices for the consumer to choose between. Therefore there exist needs for faster implementation. Because of that, this thesis will be focusing on the reduction of solve time for the scheduling of smart home appliances. Outline of this chapter: First motivating reasons for the need of scheduling of smart home appliances are presented in section 2.1. In section 2.2 information about the electricity spot price tariff and the CO₂ footprints are presented and in section 2.3 the smart home appliances technical and operation specifications are presented and described. Finally the problem formulation of the scheduling of smart home appliances is presented in section 2.4.
2.1 Motivating Reasons for Automatic Scheduling of Smart Home Appliances

The electricity price varies on an hourly basis where the price typically rises when there is a high power demand. Appliances account for about 13% of a household's electricity use [3]. There is an economic benefit that one would like to balance the load to reduce the peak electricity usage. This can be done by scheduling home appliances with controllable loads. Controllable loads are loads which one can control (i.e., by choosing the start time for a specific process or to delay its operations).

The increased awareness of the rising costs are pointed out in [4] due to the increase in electricity price. In addition to that, as mentioned before the environmental awareness continues to rise and more people start to engage themselves to live more environmentally friendly. This gives rise to the problem of balancing loads to reduce the peak electricity use and at the same time reduce the CO$_2$ emission.

Residential consumer are mostly used to a fixed electricity spot price tariff and it’s not realistic for the consumer to always keep track of the electricity spot price tariff and the CO$_2$ footprint to schedule the appliances. In [5] the benefits and savings by having demand response programs to reduce the peak electricity usage are discussed and the system Yupik is introduced to automatic help users respond to real-time electricity prices. For the Stockholm Royal Seaport project automatic scheduling algorithms will be implemented in smart home control devices which will suggest a set of different economic and environmental beneficial scheduling schemes for the smart home appliances to the consumer.

2.2 Electricity spot price tariff and CO$_2$ footprint

Hourly electricity price data tariffs for Sweden can be found at Nord Pool Spot [6]. In the Stockholm Royal Seaport project one would like to minimize the electricity bill and the CO$_2$ emissions. A typical electricity spot price tariff reaches its peak value during the day and its lowest value during the night. It’s convenient to believe that there is a somewhat fixed relation between the consumption of electricity and the emission of CO$_2$ throughout the whole day in Sweden. However as Sweden is importing more electricity during the night than during the day from Denmark, Germany, Norway, Finland and Poland [8] the CO$_2$ footprint from the consumed electricity will in general be higher during the night in comparison to during the day. This is due to that the imported electricity from Denmark, Germany, Finland and Poland mainly uses combustive fuels for electricity production [9] which is shown in the Table 2.1.
Table 2.1: Electricity Statistics from IEA (International Energy Agency) for 2011.

<table>
<thead>
<tr>
<th></th>
<th>Sweden</th>
<th>Denmark</th>
<th>Germany</th>
<th>Norway</th>
<th>Finland</th>
<th>Poland</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustive Fuels</td>
<td>11.3%</td>
<td>70.7%</td>
<td>64.7%</td>
<td>3.9%</td>
<td>50.0%</td>
<td>96.2%</td>
</tr>
<tr>
<td>Nuclear Power</td>
<td>39.7%</td>
<td>0%</td>
<td>16.1%</td>
<td>0%</td>
<td>31.5%</td>
<td>0%</td>
</tr>
<tr>
<td>Hydro Power</td>
<td>44.8%</td>
<td>0%</td>
<td>3.8%</td>
<td>95.0%</td>
<td>17.4%</td>
<td>2.0%</td>
</tr>
<tr>
<td>Other</td>
<td>4.2%</td>
<td>29.3%</td>
<td>15.4%</td>
<td>1.1%</td>
<td>1.1%</td>
<td>1.8%</td>
</tr>
</tbody>
</table>

Unlike the electricity spot price tariffs which is for an instance provided by [6] there exists no similar data online for CO$_2$ footprint. The CO$_2$ footprints are in this thesis obtained from the Institution of Ecology at the Royal Institute of Technology [7]. The electricity price tariff and the CO$_2$ footprint for 5th of January 2010 provided by Nord Pool and the Institution of Ecology at the Royal Institute of Technology is shown in Figure 2.1. As the electricity spot price is lower during the night than the day, an economic choice would be to schedule the home appliances to run during the night. However if one would also take the CO$_2$ footprint into

Figure 2.1: Electricity price tariff and CO$_2$ footprint for 5th of January 2010
account, the scheduling of the appliances is not always obvious as the CO$_2$ footprint are usually highest during the night. In other words, the scheduling of the home appliances becomes a quite complex problem with potential for economic benefits and reducing environmental impacts.

### 2.3 Appliances Technical and Operation Specifications

A smart home appliance could for an instance be a dishwasher or a washing machine. Each appliance’s operation process is divided into a set of energy phases which are interruptible sub-tasks of the appliance’s operation process. This is illustrated for a washing machine from Electrolux in Figure 2.2.

![Figure 2.2: Operation process of a washing machine from Electrolux divided into its energy phases](image)

Each energy phase consumes a specific amount of energy and requires a specific amount of time to finish its task. In addition, energy phase cannot begin until its preceding energy phase has completed its task. In other words, the energy phases for each appliance are sequential. Even though the energy phases must start after each other it does not mean that they have to start right after the preceding energy phase has finished. However neither can the next energy phase start too late, therefore there is a pre-specified upper limit for each energy phase to delay its commence (e.g. the 1st rinse of a washing machine must start within 10 minutes after the cooling has finished). Finally there are upper and lower power assignment bounds.
2.4 SCHEDULING PROBLEM AND TRADE-OFF ANALYSIS

that define the maximum and the minimum power assignment during each time instance for each energy phase.

2.4 Scheduling Problem and Trade-off Analysis

This thesis will focus on the economic benefits (minimizing the electricity bill), reduction of environmental impacts (reduction of CO$_2$ emissions) and fast implementation by scheduling of smart home appliances. This can be formulated as

\[
\text{minimize} \quad \text{Electricity bill and CO}_2 \text{ emissions} \\
\text{subject to} \quad \text{Appliance pre-specific constraints} \\
\text{Appliance-level constraints and user preferences}
\]

Two scheduling problem formulations will be introduced to minimize the electricity bill and the CO$_2$ emissions. Trade-offs such as flexibility, savings and implementation speed of these two formulations will be evaluated. To be able to minimize the electricity bill and the environmental impact one has to decide the optimal scheduling of the smart home appliances, in other words when one should assign the power to the energy phases in their respective appliance. These decisions are made with respect to pre-specified constraints from the manufacturer such as sequential processing and between-phase delay as well as appliance-level constraints (e.g. a dryer cannot be run until the washing machine has finished its tasks) and user preferences such as when the appliance should have completed all its tasks. The trade-off between electricity cost and the CO$_2$ emissions is studied through a Pareto frontier exploration. A Pareto frontier is illustrated in Figure 2.3. This method attempts to find all Pareto efficient points. A Pareto efficient point is a point that is not strictly dominated by any other [11]. A point "A" strictly dominates a point "C" if all the parameters of point "A" is not strictly greater than the corresponding parameters in C. The points along the red Pareto line are Pareto efficient. Point "C" is not Pareto efficient as it is strictly dominated by the points "A" and "B" while these are not strictly dominated by any other point, therefore point "A" and "B" are Pareto efficient points. The Pareto efficient points will hence become the set of points where one could analyse the trade-off between the electricity cost and the CO$_2$ emission. From the Pareto frontier one can then evaluate economic choices, environmental choices and balanced choices. The Pareto optimal points are obtained by solving scheduling problems whose objective functions are scalar valued, which are the weighted sum of electricity cost and CO$_2$ emission. In the following chapter the methods of the approaches will be described and defined.
Figure 2.3: Example of a Pareto frontier
Scheduling Problem Formulations

In this chapter the scheduling problem formulations for the scheduling of the smart home appliances will be introduced. Both formulations are based on mixed integer linear programming (MILP) which minimizes a linear objective function subject to linear constraints with both discrete and continuous decision variables. The first formulation, suggested by [2] is time slot based where the appliances execution period is discretized into uniform time slots. The scheduling of the smart home appliances were only optimized with respect to the electricity price. Therefore, an extension suggested by [2] to include the CO$_2$ footprint is evaluated as the first step of this thesis. The scheduling of smart home appliances then becomes a multi-objective optimization problem which is studied through a Pareto frontier. In the second formulation the execution period is not discretized. The energy phases are considered as "energy blocks" where the only scheduling decision variables are the start times of the phases. By assuming piecewise constant electricity tariff and CO$_2$ footprint, the decision problem becomes a minimization of a piecewise linear function objective functions. In the following sections the formulation of both approaches will be presented. Pre-specified manufacture constraint as well as appliance-level constraints will be described and enforced. Finally a variant of the Ramer-Douglas-Peucker algorithm will be introduced and applied for faster implementation for the piecewise linear function based formulation.

3.1 Time Slot Based Formulation

Is this approach the appliances execution period is discretized into time slots. Each appliances operation process is divided into a set of sequential energy phases which are specified by the appliances. The problem setup along with constraints and the MILP formulation is presented in the following sections.
3.1.1 Time Slot Based Problem Setup

The appliances execution time is discretized into $m$ time slots. The number of appliances are denoted $N$, and for each $i = 1, 2, \ldots, N$ the value $n_i$ denotes the number of energy phases for appliance $i$. The power assigned to energy phase $j$ in appliance $i$ over the whole period of time slot $k$ is denoted by $p_{ij}^k$. The electricity spot price is denoted by $c^k$ for time slot $k$, the CO$_2$ footprint is denoted by $d^k$ for time slot $k$ and the weighting parameter on the CO$_2$ footprint is denoted $\alpha$. From [2] the extended cost function for the MILP optimization which includes the CO$_2$ can be scalarized as a problem minimizing a scalar weighted sum of electricity cost and CO$_2$ emission

\[
\sum_{k=1}^{m} \left( c^k + \alpha d^k \right) \sum_{i=1}^{N} \sum_{j=1}^{n_i} p_{ij}^k \tag{3.1}
\]

where $\alpha$ is a given parameter. A large value on $\alpha$ means that one puts a high weight on the CO$_2$ footprint and thus increase its importance in the cost function for the scheduling of the smart home appliances. There are several constraints that needed to be enforced such that the appliances are scheduled correctly with respect to its specifications. The constraints are divided into two groups, energy constraints and timing constraints. The following imposed constraints are from [2].

**Energy Constraints**

**Energy phase energy requirement** from the appliances specifications ensures that the energy phases fulfil their energy requirements. This is imposed by the following constraint:

\[
\sum_{k=1}^{m} p_{ij}^k = E_{ij}, \quad \forall i,j \tag{3.2}
\]

where $E_{ij}$ is the energy requirements for energy phase $j$ in appliance $i$.

**Instantaneous energy phase power assignment bounds** models whether an energy phase is being processed during time slot $k$ and the lower and upper limits of the power assignment to the phase. This is imposed by the following constraint:

\[
P_{ij}^k x_{ij}^k \leq p_{ij}^k \leq P_{ij}^k x_{ij}^k, \quad \forall i,j,k \tag{3.3}
\]

where $x_{ij}^k$ is a binary decision variable $x_{ij}^k \in \{0, 1\}, \forall i,j,k$ where $x_{ij}^k = 1$ if and only if for appliance $i$, energy phase $j$ is being processed during time slot $k$. Here $P_{ij}^k$ and $P_{ij}^k$ are the appliance specific data characterizing the lower and upper limits of power assignment to the energy phases. If $x_{ij}^k = 0$, then the inequalities above collapse to a single conditions $p_{ij}^k = 0$. 
3.1. TIME SLOT BASED FORMULATION

The power safety constraint is imposed as the following constraint

$$\sum_{i=1}^{N} \sum_{j=1}^{n_i} p_{ij}^k \leq \text{PEAK}^k, \quad \forall k$$  (3.4)

where \(\text{PEAK}^k\) is the upper total slot energy bound which is provided by the external power grid operator.

Timing Constraints

Energy phase process time limits are the limits of the energy phases process times which is imposed as the following constraint

$$T_{ij} \leq \sum_{k=1}^{m} x_{ij}^k \leq T_{ij}, \quad \forall i,j$$  (3.5)

where \(T_{ij}\) and \(T_{ij}\) are the lower and upper limits of the number of time slots for energy phase \(j\) in appliance \(i\) to be processed.

Uninterruptible operation means that an energy phase, once started must be continuously processed until it is finished. This can be modelled by imposing the following constraints

$$x_{ij}^k \leq 1 - s_{ij}^k, \quad \forall i,j,k$$  (3.6a)

$$x_{ij}^{k-1} - x_{ij}^k \leq s_{ij}^k, \quad \forall i,j,\forall k = 2,3,\ldots,m$$  (3.6b)

$$s_{ij}^{k-1} \leq s_{ij}^k, \quad \forall i,j,\forall k = 2,3,\ldots,m$$  (3.6c)

where \(s_{ij}^k\) is a binary decision variable denoted \(s_{ij}^k \in \{0,1\}, \forall i,j,k\) where \(s_{ij}^k = 1\) if and only if in appliance \(i\), energy phase \(j\) is finished by time slot \(k\). For all \(i\) and \(j\), \(x_{ij}^k = 0\) if there exists a time slot \(k < k\) for which \(x_{ij}^k = 1\) and \(x_{ij}^{k+1} = 0\). In constraint (3.6a) if \(s_{ij}^k = 1\) then \(x_{ij}^k = 0\) as during time slot \(k\) energy phase \(j\) in appliance \(i\) has already finished. The binary decision variable \(s_{ij}^k\) is equal to 1 when \(x_{ij}^k\) switch from 1 to 0, in other words when the energy phase is just finished which is the situation in (3.6b). Finally the constraint (3.6c) imposes that if the process is finished at time slot \(k - 1\), then it is also finished at time slot \(k\).

Sequential Processing of the energy phases of an appliance, in other words its preceding phase has to finish before the next one can start. The sequential processing between energy phases is imposed by the following constraint

$$x_{ij}^k \leq s_{i(j-1)}^k, \quad \forall i,k,\forall j = 2,3,\ldots,n_i$$  (3.7)
an appliance-level sequential processing constraint, in other words some appliance has to finish before another one can start (e.g. washing machine has to finish its tasks before the dryer can start) is imposed by

$$x_{i_1}^k \leq s_{\tilde{i}_{n_1}}^k, \quad \forall k$$  \hspace{1cm} (3.8)

where \(\tilde{i}\) is the index of the appliance which must be finished before \(i\) can start and \(n_1\) is the last phase of appliance \(i\). In (3.7) if the preceding energy phase \(j - 1\) in appliance \(i\) has not finished at time slot \(k\), in other words \(s_{i_{(j-1)}}^k = 0\) the next energy phase \(j\) in appliance \(i\) cannot start in time slot \(k\) as \(x_{ij}^k = 0\). For (3.8) if \(s_{\tilde{i}_{n_1}}^k = 0\) the preceding appliance \(\tilde{i}\) has not finished during time slot \(k\) and the next appliance \(i\) cannot start during time slot \(k\), in other words \(x_{i_1}^k = 0\).

**Between-phase delay** is the allowed transition time between the phases \(j\) in appliance \(i\) which is imposed by the following constraints

$$\underline{D}_{ij} \leq \sum_{k=1}^{m} t_{ij}^k \leq \overline{D}_{ij}, \quad \forall j = 2, 3, \ldots, n_i$$  \hspace{1cm} (3.9)

where \(t_{ij}^k\) is a binary decision variable denoted \(t_{ij}^k \in \{0, 1\}\forall i, j, k\). Here \(\underline{D}_{ij}\) and \(\overline{D}_{ij}\) are appliance technical specifications describing the lower and upper between phase delay bounds.

$$t_{ij}^k = s_{i_{(j-1)}}^k - (x_{ij}^k + s_{ij}^k), \quad \forall j = 2, 3, \ldots, n_i$$  \hspace{1cm} (3.10)

where \(t_{ij}^k = 1\) if and only if the appliance \(i\) has finished processing energy phase \(j - 1\) and is waiting to process the energy phase \(j\).

**User time preference** is the constraint specified by the user which will decide a time interval a particular appliance should start and finish which is imposed by the following constraint

$$x_{ij}^k \leq TP_{i}^k, \quad \forall i, j, k$$  \hspace{1cm} (3.11)

where \(TP_{i}^k\) is the user specified time interval where \(TP_{i}^k = 0\) if and only if no energy phases in appliance \(i\) can be processed during time slot \(k\).

**Time Slot Based MILP formulation**

The MILP formulation can then be formulated as

minimize \quad \text{cost function (3.1)}  \hspace{1cm} (3.12)

subject to \quad \text{constraints (3.2)-(3.11)}  \hspace{1cm} (3.13)

this MILP scheduling problem is then solved using CPLEX and the YALMIP [13] interface in MATLAB [14].
3.2 Piecewise Linear Function Formulation

Each energy phase has an individual piecewise linear cost function which is computed from the appliances specifications, in other words their required energy, $E_{ij}$ and the operating time, $T_{ij}$ which are the energy and the time required to finish the energy phase’s task. In the following section the description of how to compute the piecewise linear functions will be presented as well as the problem setup along with the constraints and the MILP formulation. Finally a variant of the Ramer-Douglas-Peucker algorithm will be introduced and applied for faster implementation of the piecewise linear function approach.

3.2.1 Piecewise Linear Function

The energy phases are considered as 'energy blocks’ where the only scheduling decision variable is the start time of the energy phase and the execution period of the appliance is not discretized as in the previous formulation. The electricity spot price tariff and the CO₂ footprint are assumed to be piecewise constant which will make the objective function piecewise linear. A piecewise linear function is fully characterised by its breakpoints. A breakpoint is defined as the point when the slope of the piecewise linear curve alters. The piecewise linear functions can be computed by integrating the electricity spot price tariff or the CO₂ footprint where the domain of integration can be considered as a 'sliding block’ on the planning time horizon. Each energy block corresponds to an energy phase $j$ in appliance $i$. The sliding of an energy block with its piecewise linear function and breakpoints are illustrated in Figure 3.1.

In Figure 3.1, $A_{ij}^a$ denotes the time axis coordinate of breakpoint $a = 1, 2, 3, \ldots, q_{ij}$ where $q_{ij}$ is the number of breakpoints for energy phase $j$ in appliance $i$ and $s$ is the starting time for the energy block. Depending of the energy phase execution time $T_{ij}$ the width of the energy block will vary and therefore span over different spot prices $c^K$ for $K = 1, 2, 3, \ldots, 24$. In the same figure $F_{ij}$ is the piecewise linear function for the electricity price for energy phase $j$ in appliance $i$ and $\tau^K$ is the ending hour for spot $K$. When the operating time $T_{ij}$ spans over at most two spots their piecewise linear function can be describe with the following equations for electricity price and the CO₂ footprint

$$F_{ij}(s) = \frac{E_{ij}}{T_{ij}}(c^{ss}(\tau^{ss} - s) + c^{es}(T_{ij} + s - \tau^{ss}))$$

$$C_{ij}(s) = \frac{E_{ij}}{T_{ij}}(d^{ss}(\tau^{ss} - s) + d^{es}(T_{ij} + s - \tau^{ss}))$$

where $C_{ij}$ is the piecewise linear function for the CO₂ footprint for energy phase $j$ in appliance $i$, $c^{ss}$ and $d^{ss}$ are the spots $K$ where the energy block starts, $c^{es}$ and $d^{es}$ are...
are the spots where the energy block has finished its task after the execution time $T_{ij}$. If the energy phase at some point instead spans over more than two spots the piecewise linear function for the electricity price and the CO$_2$ footprint are instead described by the following equations

$$F_{ij}(s) = \frac{E_{ij}}{T_{ij}} \left( c^{ss}(\tau^{ss} - s) + c^{ss+1}(\tau^{ss+1} - \tau^{ss}) + \ldots + c^{es}(T + s - \tau^{es-1}) \right) \quad (3.16)$$

$$F_{ij}(s) = \frac{E_{ij}}{T_{ij}} \left( d^{ss}(\tau^{ss} - s) + d^{ss+1}(\tau^{ss+1} - \tau^{ss}) + \ldots + d^{es}(T + s - \tau^{es-1}) \right) \quad (3.17)$$

As the piecewise linear function is fully characterized by its breakpoints, the only starting times of interest are where there is a breakpoint. These fully characterized piecewise linear function are denoted $F_{ij}^a$ for electricity price and $C_{ij}^a$ for CO$_2$ footprint at breakpoint $a$. The sliding of an energy block is illustrated in Figure 3.2 for 2010-01-05 with its electricity spot price tariff and CO$_2$ footprint.

### 3.2.2 Piecewise Linear Function Problem Setup

With the calculated piecewise linear cost functions $F_{ij}^a$ and $C_{ij}^a$ one can now formulate a MILP to calculate the optimal scheduling of the smart home appliances. If the piecewise linear function $F_{ij}$ by the points $(A_{ij}^a, F_{ij}(A_{ij}^a))$ for $a = 1, 2, \ldots, q_{ij}$ on
the interval \([A_{ij}^1, A_{ij}^{q_{ij}}]\), then the start time \(s_{ij}\) can be expressed as
\[
 s_{ij} = \sum_{a=1}^{q_{ij}} \lambda_{ij}^a A_{ij}^a, \quad \forall i, j
\]  
(3.18)

where \(\lambda_{ij}^a\) is a non-negative scalar coefficient that at most two consecutive coefficients \(\lambda_{ij}^a\) can be non-zero for each energy phase \(j\) in appliance \(i\). This gives the choice of any \(\lambda_{ij}^a\) a unique starting time \(s_{ij}\). To this effect an binary variable \(y_{ij}^a \in \{0, 1\}, \forall i, j, a\) and the following constraints are introduced [12]
\[
 \sum_{a=1}^{q_{ij}} \lambda_{ij}^a = 1, \quad \forall i, j, a
\]  
(3.19)

\[
 \lambda_{ij}^1 \leq y_{ij}^1, \quad \forall i, j
\]  
(3.20)

\[
 \lambda_{ij}^a \leq y_{ij}^{a-1} + y_{ij}^a, \quad \forall i, j, \forall a = 2, 3, \ldots, q_{ij} - 1
\]  
(3.21)
\[
\lambda_{ij} \leq y_{ij}^{q_j - 1}, \quad \forall i, j \\
\sum_{a=1}^{q_j - 1} y_{ij}^a = 1, \quad \forall i, j \\
\lambda_{ij}^a \geq 0, \quad \forall i, j, a
\]

The cost function to minimize is then formulated to

\[
N \sum_{i=1}^{N} \sum_{j=1}^{n_i} q_{ij} \sum_{a=1}^{n} \left( F_{ij}^a + \alpha C_{ij}^a \right) \lambda_{ij}^a
\]

The pre-specified manufacture constraints as well as appliance-level constraints and user preferences are described in the following sections.

Constraints

The Sequential processing and Between-phase delay, i.e. that an energy phase cannot start until the preceding phase has finished its task is imposed by following constraint

\[
s_{ij} + T_{ij} \leq s_{i(j+1)} \leq s_{ij} + T_{ij} + D_{ij}, \quad \forall i, \forall j = 1, 2, \ldots, n_i - 1
\]

where \(D_{ij}\) is the between-phase delay specified by the appliances specifications for each energy phase \(j\) in appliance \(i\). One would preferable want to schedule some appliances before others, e.g. the dryer should be run after the washing machine has finished its tasks. This is imposed by the following constraint

\[
s_{i\tilde{n_i}} + T_{i\tilde{n_i}} \leq s_{i1}
\]

where \(\tilde{i}\) is the index of the appliance which must be finished before appliance \(i\) can start and \(n_{i}\) is the last phase of appliance \(i\).

The user time preferences which is the user specified interval for which the appliance should be run. This is imposed by the following constraints

\[
s_{i1} \geq TP_{i}^{\text{start}}
\]

\[
s_{i\tilde{n_i}} \leq TP_{i}^{\text{end}}
\]

where \(TP_{i}^{\text{start}}\) is earliest allowed starting time for the appliance \(i\) and \(TP_{i}^{\text{end}}\) is the latest time which the appliance \(i\) has finished its task.
Piecewise Linear Function MILP Formulation

The MILP formulation can now be formulated as

\[
\begin{align*}
\text{minimize} & \quad \text{cost function (3.25)} \\
\text{subject to} & \quad \text{constraints (3.19)-(3.21) and (3.26)-(3.29)}
\end{align*}
\]

This MILP scheduling problem is then solved using CPLEX and the YALMIP interface in MATLAB. In this formulation the power assignment is not as flexible as the previous formulation, as there is a constant power assignment for each energy phase throughout its execution time. Therefore there is no constraint to limit the instantaneous energy phase power assignment bounds. In addition, a power safety constraint couldn’t be enforced in the piecewise linear formulation. Even though the piecewise linear function formulation is flexible in the sense that its not limited by time slot lengths, there exists drawbacks that will be evaluated in the following chapters.

3.2.3 Reduction of Breakpoints Using a variant of the Ramer-Douglas-Peucker Algorithm

The main idea behind the Ramer-Douglas-Peucker algorithm is to reduce the number of points in a curve to a similar curve with fewer points where the simplified curve consists of a subset of the points of the original curve. The algorithm divides the curve recursively and keeps the function values at the kept breakpoints. The rest of the function values are based on linear interpolation using the closest breakpoints. The number of breakpoints is reduced while guaranteeing that the error is bounded from above [15] [16]. In this thesis a variant of the Ramer-Duglas-Peucker algorithm is used where the Hausdorff distance error is replaced with the maximum absolute functions difference to adapt to simplification of a piecewise linear function instead of simplifying a planar curve. The algorithm is illustrated in Figure 3.3. The recursive algorithm starts by saving the starting point and end point of the piecewise linear curve. It then makes a linear interpolation between the saved points and then finds the point furthest from the interpolated curve. If the difference error is equal or smaller to the allowed upper bound the point can be discarded but if the difference error is higher than the allowed upper bound, the point must be kept and is then saved. The algorithm then recursively recall itself with the saved points and again an linear interpolation is made and the point furthest away from the interpolated curve is evaluated (see step 1-4 in Figure 3.3).
Figure 3.3: Smoothing a piecewise linear curve with the Ramer-Douglas-Peucker algorithm.
Results from different scheduling scenarios will be presented in the following sections. Each scenario will be presented and described in the beginning of each new section. First the appliance manufacture pre-specifications are given in Tables 4.1-4.3 for a dishwasher, a washing machine and a dryer respectively [10]. The listed 'Energy' column in the tables is the required energy $E_{ij}$ in (3.2) and (3.14)-(3.17), 'Min power' and 'Max power' are the lower and upper limits for energy assignment in each slot, $P_{ij}^k$ and $P_{ij}^k$ in (3.3). Finally the 'Nom. exe. time' is the nominal execution time $T_{ij}$.

<table>
<thead>
<tr>
<th>Energy Phase</th>
<th>Energy</th>
<th>Min power</th>
<th>Max power</th>
<th>Nom. exe. time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-wash</td>
<td>16.0 Wh</td>
<td>6.47 W</td>
<td>140 W</td>
<td>14.9 min</td>
</tr>
<tr>
<td>Wash</td>
<td>751.2 Wh</td>
<td>140.26 W</td>
<td>2117.8 W</td>
<td>32.1 min</td>
</tr>
<tr>
<td>1st rinse</td>
<td>17.3 Wh</td>
<td>10.28 W</td>
<td>132.4 W</td>
<td>10.1 min</td>
</tr>
<tr>
<td>Drain</td>
<td>1.6 Wh</td>
<td>2.26 W</td>
<td>136.2 W</td>
<td>4.3 min</td>
</tr>
<tr>
<td>2st rinse</td>
<td>572.3 Wh</td>
<td>187.3 W</td>
<td>2143 W</td>
<td>18.3 min</td>
</tr>
<tr>
<td>Drain &amp; dry</td>
<td>1.7 Wh</td>
<td>0.2 W</td>
<td>2.3 W</td>
<td>52.4 min</td>
</tr>
</tbody>
</table>

The between-phase delay $D_{ij}$ is assumed zero for all phases and $D_{ij}$ is assumed to be 5, 10 and 0 minutes for the energy phases in the dishwasher, washing machine and dryer respectively. User preferences are assumed to be that the dishwasher should run between 7pm and the end of the day, the washing machine and the dryer should run anytime between 9am and 11pm if no other user preferences are stated. However, the washing machines tasks have to finish before the dryer can start. For the time slot based formulation there are additional constraints that have to be defined such as the peak signal (total slot energy upper bond). The peak signal is assumed to be 5500 Wh and the upper and lower operation times $T_{ij}$ and $T_{ij}$ are assumed to be between 80% and 120% of the nominal operation time.
### Table 4.2: Washing machine technical specifications.

<table>
<thead>
<tr>
<th>Energy Phase</th>
<th>Energy</th>
<th>Min power</th>
<th>Max power</th>
<th>Nom. exe. time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Movement</td>
<td>118 Wh</td>
<td>27.231 W</td>
<td>2100 W</td>
<td>26 min</td>
</tr>
<tr>
<td>Pre-heating</td>
<td>5.5 Wh</td>
<td>5 W</td>
<td>300 W</td>
<td>6.6 min</td>
</tr>
<tr>
<td>Heating</td>
<td>2054.9 Wh</td>
<td>206.523 W</td>
<td>2200 W</td>
<td>59.7 min</td>
</tr>
<tr>
<td>Maintenance</td>
<td>36.6 Wh</td>
<td>11.035 W</td>
<td>200 W</td>
<td>19.9 min</td>
</tr>
<tr>
<td>Cooling</td>
<td>18 Wh</td>
<td>10.8 W</td>
<td>500 W</td>
<td>10 min</td>
</tr>
<tr>
<td>1st rinse</td>
<td>18 Wh</td>
<td>10.385 W</td>
<td>700 W</td>
<td>10.4 min</td>
</tr>
<tr>
<td>2nd rinse</td>
<td>17 Wh</td>
<td>9.903 W</td>
<td>700 W</td>
<td>10.3 min</td>
</tr>
<tr>
<td>3rd rinse</td>
<td>78 Wh</td>
<td>23.636 W</td>
<td>1170 W</td>
<td>19.8 min</td>
</tr>
</tbody>
</table>

### Table 4.3: Dryer technical specifications.

<table>
<thead>
<tr>
<th>Energy Phase</th>
<th>Energy</th>
<th>Min power</th>
<th>Max power</th>
<th>Nom. exe. time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drying</td>
<td>2426.3 Wh</td>
<td>120.51 W</td>
<td>1454 W</td>
<td>120.8 min</td>
</tr>
</tbody>
</table>

For both formulations Pareto frontiers will be presented to study the trade-offs between the economic benefits and the environmental benefits as well as balanced choices where one achieves a good trade-off between reducing the electricity bill and the CO$_2$ emissions at the same time. The most economic choice corresponds to the weighting parameter being $\alpha = 0$, in that way the CO$_2$ footprint will not be taken into account in (3.1) and (3.18). On the other hand, for the environmental choice the parameter $\alpha$ would take a very large value. The balanced choice is derived by looking at each unique point in the Pareto frontier. The points are evaluated by looking at its two parameters, the electricity cost and CO$_2$ emission. For the parameter, the values around their respective median are evaluated. Finally a balanced point is chosen whose parameters are within the range of the evaluated values around the median for both parameters. There is no perfectly balanced choice of the weighting parameter $\alpha$ such that the trade-off between economic benefits are equal to the environmental benefits. Results from scalability test, additional simulation days and simulations when excluding the user preferences will be presented in the following sections for both formulations. The electricity tariff and CO$_2$ footprint for the two additional days are presented in Figures 4.1 and 4.2.

For the piecewise linear function based formulation the results of applying a variant of the Ramer-Douglas-Peucker algorithms will also be presented. Finally trade-offs between the formulations will be presented where one look at the savings and implementation speed.
4.1 Time Slot Based Formulation

4.1.1 Reduction of Electricity Bill and CO$_2$ Emissions

The optimal reduction of electricity bill and CO$_2$ emissions for 5th of January 2010 are solved for (3.12) and (3.13) and are studied through the Pareto frontier shown in Figure 4.3. The Pareto frontier shows that there are economic choices, environmental choices as well as balanced choices. The weighted sum approach does not guarantee finding all Pareto optimal solutions, therefore the weighted sum approach can lead to the same Pareto optimal solution. This is observed in the Pareto frontier when the Pareto efficient points are clustering. The numerical values for these choices are presented in the Table 4.4.

These results shows that one can achieve economic benefits and at the same time reduce the environmental impacts by reduction of CO$_2$ emissions. The price saving and CO$_2$ reduction are with respect to the worst case scheduling for a balanced choice, in other words when one solve the scheduling problem where the objective function is maximized instead of minimized. For the balanced choice the weighting

---

**Figure 4.1:** Electricity price tariff and CO$_2$ footprint for 11th of February 2010.
CHAPTER 4. RESULTS

Figure 4.2: Electricity price tariff and CO$_2$ footprint for 12th of October 2010.

Table 4.4: Economic benefits and environmental benefits for 3 appliances with 5 min time slots for 2010-01-05 with respect to worst case scheduling when $\alpha = 9$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Min price</th>
<th>Price Saving</th>
<th>Min CO$_2$</th>
<th>CO$_2$ Red.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Economic)</td>
<td>3.046 SEK</td>
<td>37.50%</td>
<td>0.385 kg</td>
<td>11.72%</td>
</tr>
<tr>
<td>9 (Balanced)</td>
<td>3.124 SEK</td>
<td>35.90%</td>
<td>0.364 kg</td>
<td>16.51%</td>
</tr>
<tr>
<td>40 (Environmental)</td>
<td>3.127 SEK</td>
<td>35.83%</td>
<td>0.364 kg</td>
<td>16.51%</td>
</tr>
</tbody>
</table>

parameter $\alpha = 9$ which gives the maximum price of 4.873 SEK and maximum CO$_2$ emission of 0.436 kg for 2010-01-05. Here one can see that the "Balanced" choice and "Environmental" choice gives the same reduction in terms of percentage, but it’s good to notice that it would change if each choice were with respect to its own unique worst case scheduling. By comparing the choices with the same worst case scheduling one can instead more intuitive compare the different choices. The sum of energy assigned for each time slot for the "Balanced" choice with the electricity price tariff and the CO$_2$ footprint are shown in Figure 4.4. The power profiles for each appliance for the well-balanced case when $\alpha = 9$ is shown in Figure 4.5.
4.1. **TIME SLOT BASED FORMULATION**

The choices will either benefit an economic thinking or an environmental thinking more than the other. There is one weight parameter $\alpha$ for which the choice is most economic beneficial and one for which it is most environmental beneficial. The most economic choice is when the weighting parameter $\alpha = 0$ as then the cost function (3.1) will only be optimized with respect to the price. The most environmental beneficial choice can be determined by excluding the electricity price, $c^k$ from (3.1). Then one can show that the most environmentally scheduling is the same as when one optimize with respect to both electricity price and CO$_2$ footprint when $\alpha = 40$ for 2010-01-05. This is shown in Table 4.5.

**Figure 4.3:** Pareto frontier for 3 appliances with time slot 5 minutes for 2010-01-05 with $\alpha$ ranging from 0 to 40 with 0.1 steps.

<table>
<thead>
<tr>
<th>Min CO$_2$ emissions</th>
<th>Saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.364 kg</td>
<td>16.51%</td>
</tr>
</tbody>
</table>
CHAPTER 4. RESULTS

Figure 4.4: Total energy assignment for each time slot and the electricity price tariff and CO₂ footprint for 2010-01-05 with $\alpha = 9$.

4.1.2 Excluding the user preferences

For these results the user preferences are removed, in other words they are allowed to run any time from 12 am to the end of the day. The optimal reduction of electricity bill and CO₂ emissions are studied in the Pareto frontier shown in Figure 4.6. From the Pareto frontier one can determine that there are economic choices, environmental choices as well as balanced choices. Numerical values for these choices are presented in Table 4.6. The price saving and the reduction of CO₂ emissions are with respect to the balanced choice when $\alpha = 8$ which gives the maximum price of 5.126 SEK and maximum CO₂ emission of 0.537 kg for 2010-01-05 when excluding the user preferences.

This shows that when the appliances are allowed to run during the night one can achieve higher economic benefits by increasing the CO₂ emissions as a pay-off. In addition one can as well reduce the CO₂ emission more by increasing the electricity bill as a pay-off. A balanced choice when $\alpha = 8$ shows that one can achieve a higher price savings and at the same time reduce the CO₂ emissions if the appliances are
4.1. TIME SLOT BASED FORMULATION

Figure 4.5: Power profiles for appliances with $\alpha = 9$ for 2010-01-05.

Table 4.6: Economic benefits and environmental benefits for 3 appliances with 5 min time slots with extended user preferences for 2010-01-05 with respect to worst case scheduling when $\alpha = 8$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Min price</th>
<th>Price Saving</th>
<th>Min $\text{CO}_2$</th>
<th>$\text{CO}_2$ Red.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Economic)</td>
<td>2.589 SEK</td>
<td>49.49 %</td>
<td>0.636 kg</td>
<td>-18.43 %</td>
</tr>
<tr>
<td>8 (Balanced)</td>
<td>2.904 SEK</td>
<td>43.35 %</td>
<td>0.366 kg</td>
<td>31.84 %</td>
</tr>
<tr>
<td>40 (Environmental)</td>
<td>3.007 SEK</td>
<td>41.33 %</td>
<td>0.359 kg</td>
<td>33.15 %</td>
</tr>
</tbody>
</table>

allowed to run throughout the day. If one would chose the economic choice one can see that the $\text{CO}_2$ emissions increase with 18.43 %. This does not mean that you actually increase your $\text{CO}_2$ emissions by choosing an economic scheduling but rather what effect an economic scheduling could have compared to the worst case scheduling for a balanced choice.
CHAPTER 4. RESULTS

4.1.3 Scheduling for different days

The optimal scheduling is calculated with the user preferences such that the dishwasher should run between 7 pm and the end of the day, the washing machine and the dryer should run between 9 am and 11 pm where the dryer cannot start until the washing machine have finished its tasks. The optimal schedules are calculated for 11th of February 2010 and 12th of October 2010 and are studied through the Pareto frontiers shown in Figures 4.7 and 4.8. From these Pareto frontiers one can determine that there are economic choices, environmental choices as well as balanced choices. The price saving and the reduction of CO\textsubscript{2} emissions for 2010-02-11 are with respect to the balanced choice when $\alpha = 8$ which gives the maximum price of 5.578 SEK and maximum CO\textsubscript{2} emission of 0.431 kg and the price saving and the reduction of CO\textsubscript{2} emission for 2010-10-12 are with respect to the balanced choice when $\alpha = 9.5$ which gives the maximum price of 3.374 SEK and maximum CO\textsubscript{2} emissions of 0.355 kg. The numerical values for these choices are presented in the Tables 4.7 and 4.8 for 11th of February 2010 and 12th of October 2010 respectively.
4.1. TIME SLOT BASED FORMULATION

Figure 4.7: Pareto frontier for 3 appliances with time slot 5 minutes for 2010-02-11 with α ranging from 0 to 40 with 0.1 steps.

Table 4.7: Economic benefits and environmental benefits for 3 appliances with 5 min time slots for 2010-02-11 with respect to worst case scheduling when α = 8.

<table>
<thead>
<tr>
<th>α</th>
<th>Min price</th>
<th>Price Saving</th>
<th>Min CO₂</th>
<th>CO₂ Red.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Economic)</td>
<td>3.743 SEK</td>
<td>32.90 %</td>
<td>0.424 kg</td>
<td>-0.71 %</td>
</tr>
<tr>
<td>8 (Balanced)</td>
<td>3.901 SEK</td>
<td>32.50 %</td>
<td>0.393 kg</td>
<td>6.72 %</td>
</tr>
<tr>
<td>40 (Environmental)</td>
<td>4.077 SEK</td>
<td>26.91 %</td>
<td>0.385 kg</td>
<td>6.78 %</td>
</tr>
</tbody>
</table>

The results for 2010-02-11 shows that for a day when one cannot reduce the CO₂ emissions one can still save a lot in terms of electricity costs.

The results from 2010-10-12 shows that for a day when one cannot save that much money one can still reduce the CO₂ emissions.
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0.29 0.3 0.31 0.32 0.33 0.34
2.92
2.94
2.96
2.98
3
3.02
3.04
CO2 emission [kg]
Electricity price [SEK]
Environmental choice
Economic choice
Balanced choice

Figure 4.8: Pareto frontier for 3 appliances with time slot 5 minutes for 2010-10-12 with α ranging from 0 to 40 with 0.1 steps.

Table 4.8: Economic benefits and environmental benefits for 3 appliances with 5 min time slots for 2010-10-12 with respect to worst case scheduling when α = 9.5.

<table>
<thead>
<tr>
<th>α</th>
<th>Min price</th>
<th>Price Saving</th>
<th>Min CO2</th>
<th>CO2 Red.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Economic)</td>
<td>2.901 SEK</td>
<td>14.02%</td>
<td>0.338 kg</td>
<td>4.84%</td>
</tr>
<tr>
<td>9.5 (Balanced)</td>
<td>3.026 SEK</td>
<td>10.31%</td>
<td>0.283 kg</td>
<td>20.28%</td>
</tr>
<tr>
<td>40 (Environmental)</td>
<td>3.049 SEK</td>
<td>9.63%</td>
<td>0.281 kg</td>
<td>20.85%</td>
</tr>
</tbody>
</table>

4.1.4 Scalability Test

In this section the scalability of the time slot based approach will be evaluated. This is preferred as one would like to test the limit of computational and memory requirements for solving (3.12) by increasing the number of appliances for time slot lengths 5 minutes and 10 minutes with the weighting parameter α = 9 for 2010-01-05 and no user preferences. The time slot length is the length of the uniform time slots which the appliances execution period is discretized into. The appliances are divided into a set, which consists of a dishwasher, washing machine and a dryer.
4.2. PIECEWISE LINEAR FUNCTION BASED FORMULATION

The scalability test evaluates 1, 2 and 3 sets, in other words 3, 6 and 9 appliances. These results are presented in the Figure 4.9. The results shows that both the slot lengths and the number of appliances have a significant impact on the solve time.

![Graph showing scalability test](image)

**Figure 4.9:** Scalability test by increasing the number of appliances to evaluate the increase of solve time for 2010-01-05 with $\alpha = 9$ and no user preferences.

4.2 Piecewise Linear Function Based Formulation

4.2.1 Reduction of Electricity Bill and CO$_2$ Emissions

The optimal reduction of electricity bill and CO$_2$ emissions for 5th of January 2010 are studied through the Pareto frontier shown in Figure 4.10. One can conclude that there are economic choices, environmental choices as well as balanced choices. The price savings and the CO$_2$ reductions are with respect to the balanced choice when $\alpha = 15$, which gives the maximum price of 4.693 SEK and maximum CO$_2$ emission.
of 0.447 kg for 2010-01-05. The numerical values for these choices are presented in
the Table 4.9.

Table 4.9: Economic benefits and environmental benefits for 3 appliances for 2010-01-05
with respect to worst case scheduling when \( \alpha = 15 \).

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>Min price</th>
<th>Price Saving</th>
<th>Min CO(_2)</th>
<th>CO(_2) Red.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Economic)</td>
<td>3.064 SEK</td>
<td>38.26 %</td>
<td>0.379 kg</td>
<td>15.23 %</td>
</tr>
<tr>
<td>15 (Balanced)</td>
<td>3.071 SEK</td>
<td>34.56 %</td>
<td>0.376 kg</td>
<td>15.90 %</td>
</tr>
<tr>
<td>40 (Environmental)</td>
<td>3.243 SEK</td>
<td>30.90 %</td>
<td>0.370 kg</td>
<td>17.24 %</td>
</tr>
</tbody>
</table>

These results shows gives the same results as the time slot based formulation that
one can achieve both economic benefits and environmental benefits. The most en-
vironmental beneficial choice can be determined by the same procedure described
for the time slot based approach by excluding the electricity price, \( c^K \) from (3.18).

It is shown that the most environmental scheduling is the same as when one opti-
mize with respect to both electricity price and CO\(_2\) footprint when \( \alpha = 40 \). This
4.2. PIECEWISE LINEAR FUNCTION BASED FORMULATION

numerical results is shown in Table 4.10.

**Table 4.10:** Scheduling the smart home appliances with respect to only the CO\textsubscript{2} footprint for 2010-01-05.

<table>
<thead>
<tr>
<th>Min CO\textsubscript{2} emissions</th>
<th>Saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.370 kg</td>
<td>17.24%</td>
</tr>
</tbody>
</table>

4.2.2 Excluding the user preferences

For these results there are no user preferences. The optimal reduction of electricity price bill and CO\textsubscript{2} emissions are studied in the Pareto frontier shown in Figure 4.11. One can conclude that there are economic choices, environmental choices as well as balanced choices. The price saving and the reduction of CO\textsubscript{2} emissions are with respect to the balanced choice when $\alpha = 9$ which gives the maximum price of 5.122 SEK and maximum CO\textsubscript{2} emissions of 0.536 kg for 2010-01-05 with extended...
user preferences. The numerical values for these choices are presented in the Table 4.11.

**Table 4.11:** Economic benefits and environmental benefits for 3 appliances for 2010-01-05 with respect to worst case scheduling when $\alpha = 9$ with extended user preferences.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Min price</th>
<th>Price Saving</th>
<th>Min CO$_2$</th>
<th>CO$_2$ Red.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Economic)</td>
<td>2.591 SEK</td>
<td>49.41%</td>
<td>0.633 kg</td>
<td>-18.10%</td>
</tr>
<tr>
<td>9 (Balanced)</td>
<td>2.876 SEK</td>
<td>43.85%</td>
<td>0.373 kg</td>
<td>30.41%</td>
</tr>
<tr>
<td>40 (Environmental)</td>
<td>3.008 SEK</td>
<td>41.27%</td>
<td>0.363 kg</td>
<td>32.28%</td>
</tr>
</tbody>
</table>

This shows that when the appliances are allowed to run during the night one can achieve higher economic benefits by decreasing the environmental benefits as a pay-off. In addition one can as well achieve higher environmental benefits by decreasing the economic benefits as a pay-off. A balanced choice when $\alpha = 9$ shows that one can achieve a higher price savings and at the same time reduce the CO$_2$ emissions if the appliances are allowed to run throughout the day.

4.2.3 Additional simulation days

The optimal reduction of electricity bill and CO$_2$ emissions for 11th of February 2010 and 12th of October 2010 are studied through the Pareto frontiers shown in Figures 4.12 and 4.13. From these Pareto frontiers one can determine that there are economic choices, environmental choices as well as balanced choices. The price savings and the CO$_2$ reductions for 2010-02-11 are with respect to the balanced choice when $\alpha = 10$ which gives the maximum price of 5.560SEK and maximum CO$_2$ emission of 0.412 kg. The price savings and the CO$_2$ reductions for 2010-10-12 are with respect to the balanced choice when $\alpha = 7.5$ which gives the maximum price of 3.350SEK and maximum CO$_2$ emission of 0.350 kg. The numerical values for the choices are presented in the Tables 4.12 and 4.13 for 11th of February 2010 and 12th of October 2010 respectively.

**Table 4.12:** Economic benefits and environmental benefits for 3 appliances for 2010-02-11 with respect to worst case scheduling when $\alpha = 10$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Min price</th>
<th>Price Saving</th>
<th>Min CO$_2$</th>
<th>CO$_2$ Red.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Economic)</td>
<td>3.760 SEK</td>
<td>32.37%</td>
<td>0.425 kg</td>
<td>-3.16%</td>
</tr>
<tr>
<td>10 (Balanced)</td>
<td>3.921 SEK</td>
<td>29.48%</td>
<td>0.395 kg</td>
<td>4.13%</td>
</tr>
<tr>
<td>40 (Environmental)</td>
<td>4.147 SEK</td>
<td>25.41%</td>
<td>0.385 kg</td>
<td>6.55%</td>
</tr>
</tbody>
</table>

The results for 2010-02-11 shows that for a day when one cannot reduce the CO$_2$ emission that much, one can still save a lot in terms of electricity cost.

The results for 2010-10-12 shows that for a day when one cannot save in terms of electricity cost, one can still reduce the CO$_2$ emission a lot.
4.2. PIECEWISE LINEAR FUNCTION BASED FORMULATION

Figure 4.12: Pareto frontier for 3 appliances for 2010-02-11 with $\alpha$ ranging from 0 to 40 with 0.1 steps.

Table 4.13: Economic benefits and environmental benefits for 3 appliances for 2010-10-12 with respect to worst case scheduling when $\alpha = 7.5$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Min price</th>
<th>Price Saving</th>
<th>Min CO$_2$</th>
<th>CO$_2$ Red.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Economic)</td>
<td>2.918 SEK</td>
<td>12.90%</td>
<td>0.332 kg</td>
<td>5.14%</td>
</tr>
<tr>
<td>7.5 (Balanced)</td>
<td>3.002 SEK</td>
<td>10.39%</td>
<td>0.287 kg</td>
<td>18.00%</td>
</tr>
<tr>
<td>40 (Environmental)</td>
<td>3.050 SEK</td>
<td>9.00%</td>
<td>0.282 kg</td>
<td>19.43%</td>
</tr>
</tbody>
</table>

4.2.4 Scalability Test

In this section the scalability of the piecewise linear function based approach will be evaluated. This is preferred as one would like to test the limit of computational and memory requirements for solving (3.30) by increasing the number of sets of appliances with 3, 6, 9, 12, 15 and 18 appliances. This test is for 2010-01-05 with $\alpha = 15$ and no user preferences. These results are presented in Figure 4.14. From Figure 4.14 one can conclude that for the piecewise linear function based approach the implementation time increases relative little by adding number of appliances.
Figure 4.13: Pareto frontier for 3 appliances for 2010-10-12 with $\alpha$ ranging from 0 to 40 with 0.1 steps.

cmpared to the time slot based approach.

4.2.5 Ramer-Douglas-Peucker algorithm for breakpoint reduction

Figure 4.15 shows how one can reduce the solve time by reducing the number of breakpoints by allowing a 1%, 3%, 5%, 8% and 10% difference error. The results are with 3 appliances for 2010-01-05 with $\alpha = 15$ and no user preferences. One can conclude that the solve time will decrease with the proposed reduction algorithm in 3.2.3. The number of breakpoints reduced for some case are presented in the Figure 4.16 where the number of breakpoints are the sum of all breakpoints for all energy phases piecewise linear functions in each appliance. In Figure 4.17 the changes of the electricity cost and the CO$_2$ emission are presented when applying the proposed reduction algorithm. These results show that by allowing a difference error there will be a change of electricity cost and CO$_2$ emissions.
### 4.3 Comparison between the two formulations

#### 4.3.1 Reduction of electricity bill and CO$_2$ emissions

In Table 4.14 one can see that the saving for the time slot based approach is higher than for the piecewise linear function based approach for their respective balanced choice, in other words when $\alpha = 9$ for the time slot based formulation and $\alpha = 15$ for the piecewise linear function formulation.

<table>
<thead>
<tr>
<th>Method</th>
<th>Min price</th>
<th>Price Saving</th>
<th>Min CO$_2$</th>
<th>CO$_2$ Saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time slot formulation</td>
<td>3.124</td>
<td>35.90%</td>
<td>0.364 kg</td>
<td>16.51%</td>
</tr>
<tr>
<td>PWLF formulation</td>
<td>3.071</td>
<td>34.56%</td>
<td>0.376 kg</td>
<td>15.9%</td>
</tr>
</tbody>
</table>

Figure 4.14: Scalability test by increasing the number of appliances to evaluate the increase of solve time.
CHAPTER 4. RESULTS

Figure 4.15: The variant of the Ramer-Douglas-Peucker algorithm effects on the solve time by allowing 1%, 3%, 5%, 8% and 10% difference error.

The reason why the time slot formulation achieves a higher saving in electricity costs and reduction of CO$_2$ emissions is due to that the time slot based approach allows a more flexible power assignment to the energy phases, the power assigned can vary within a range while for the piecewise linear function approach the power assignment for each energy phase is constant. The solve time for the time slot formulation with 5 minutes time slot length is 6.94 seconds and the solve time for the piecewise linear formulation is 1.16 seconds which gives a almost sixfold faster implementation speed scheduling 3 appliances for 2010-01-05.

4.3.2 Implementation speed

In the Figure 4.18 the implementation speed with respect to the number of appliances for the piecewise linear function formulation and the time slot formulation with time slots equal to 5 minutes and 10 minutes for 2010-01-05 with no user preferences.

Here one can see that the piecewise linear formulation is much faster as the numbers
4.3. COMPARISON BETWEEN THE TWO FORMULATIONS

Figure 4.16: Number of breakpoints reduced using the proposed variant of the Ramer-Douglas-Peucker algorithms.

of appliances increase.
Figure 4.17: Changes of electricity cost and CO$_2$ emission when applying the proposed reduction algorithm for 3 appliances for 2010-01-15 with $\alpha = 15$ and no user preferences when allowing 1%, 3%, 5%, 8% and 10% difference error.
4.3. COMPARISON BETWEEN THE TWO FORMULATIONS

Figure 4.18: Implementation speed for the PWLF formulation with $\alpha = 15$ and time slot formulation with $\alpha = 15$ for time slots equal to 5 minutes and 10 minutes for 2010-01-05 with no user preferences.
Conclusion

This thesis showed that the trade-off analysis can be performed by study the Pareto frontier where economic, balanced and environmental scheduling choices were computed. The results showed that there are economic benefits as well as environmental benefits. How much savings depends greatly on the electricity price tariffs and the CO\textsubscript{2} footprints volatility and how much they change throughout the day as one would achieve higher saving for days when the volatility is high. A well-balanced choice where one reduce both the electricity bill and the CO\textsubscript{2} emissions for the time slot formulation were found out to be for $\alpha = 9$ where one could save up to 35.9\% of electricity costs as well as reducing the CO\textsubscript{2} emissions with up to 16.51\%, while for the piecewise linear function formulation one could save up to 34.56\% of electricity cost as well as reducing the CO\textsubscript{2} emissions with up to 15.9\% with the weighting parameter $\alpha = 15$. These savings are with respect to the worst case scheduling for respective formulation. A more economic concerned consumer should choose $\alpha = 0$ and an environmentally concerned should choose a high $\alpha$. The simulation day 2010-05-01 was a unusual cold winter day in Sweden which gave a rise to a volatile electricity price spot tariff, while for an ordinary Swedish day the savings is about 2.5\% in electricity cost, however in New York City one could save about 47\% in electricity cost [2]. In other words, this automatic scheduling algorithm could be of great use in other countries, however it has to be further evaluated.

As the time slot based formulation can assign any amount of power to each energy phase within its upper and lower power assignment bounds it will give higher savings than for the piecewise linear based formulation. However, the piecewise linear function based formulation can start during any time as its not dependent on time slots. Therefore the piecewise linear function based formulation will give a larger variation of scheduling schemes than for the time slot based formulation. This is confirmed as the Pareto frontier for the piecewise linear function based formulation have more unique points (typically twice as many) than for the Pareto frontier for the time slot based formulation with a time slot length equal to 5 minutes.

The most important results are that the piecewise linear function based formu-
lation implementation speed is sixfold faster than the time slot based formulation which resulted in similar scheduling schemes. The solve time for the piecewise linear formulation grows much slower than that of the time slot based formulation. This is because the piecewise linear function formulation requires much fewer binary decision variables to model. There are however drawbacks with the piecewise linear function based formulation. For instance, it is much harder to extend the formulation for dynamical systems (e.g. when the planning for solar cells and batteries is also considered.).

The applied variant of the Ramer-Douglas-Peucker algorithm were able to reduce the number of breakpoints which reduced the solve time for the piecewise linear formulation. The benefits of the Ramer-Douglas-Peucker algorithm become more obvious as the number of appliances increases. It showed that the electricity costs and the CO₂ emissions changes by allowing a difference error. However, there are no intuitive conclusion of how the allowed difference will effect the electricity costs and the CO₂ emissions, therefore further evaluations and analysis have to be made.

For future work extensive simulations have to be made. As this thesis only evaluates three days in 2010 extensive simulations for more days, alternatively a whole year. This is needed to be able to evaluate a more general benefit from this automatic scheduling algorithm in smart control devices. In this thesis the same set of appliances were used, it would be interesting to investigate the results by adding a wider range of different appliances. For the time slot based formulation several extensions could be made to include dynamical systems such as solar cells and batteries which are described in [2]. For the piecewise linear formulation extensions are hard to formulate, therefore further investigations have to be made.
References


[14] MATLAB, version R2010b is a registered trademark of The MathWorkds Inc..
